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# Reliability Considerations for Multiple-Spot-Beam Communication Satellites

By A. S. ACAMPORA

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*Reliability considerations associated with multiple-spot-beam satellite systems are explored. If each coverage area is serviced by a single transponder, then loss of transponders due to failure eliminates all service to the areas covered by those transponders. Thus, failures are quite costly compared to a system employing global coverage with multiple transponders, where a limited number of transponder failures results in a slight increase in the traffic demand upon the survivors. Since the total orbital weight of a satellite is fixed, any redundant hardware deployed to improve reliability reduces the number of active transponders that can be supported, and a highly efficient redundancy strategy must be employed. Cold standby redundancy with complete spare interconnectivity is studied and appropriate reliability formulas are derived. A specific satellite concept dominated by final power amplifier failures is studied in detail, and it is found, for typical failure rates, that a 27-percent reduction in capacity must be accepted to provide for a single satellite lifetime reliability of 99 percent. Various techniques for employing the in-orbit redundancy of a spare satellite are investigated to increase reliability while minimizing capacity reduction. Bounds are derived and a reliability of at least 99 percent is shown possible for a system containing three active satellites plus one spare, at a cost of 9 percent in potential capacity of the active satellites.*

## I. INTRODUCTION

Multiple-spot-beam antennas for communication satellites offer the potential for greatly increasing the traffic capacity of the satellite since the allocated frequency band can be reused in the various spot beams.[1–4] In such a configuration, we might reduce the number of required satellite transponders by allocating a single wideband transponder, consisting of a receiver and a transmitter, to each antenna port as shown in Fig. 1.

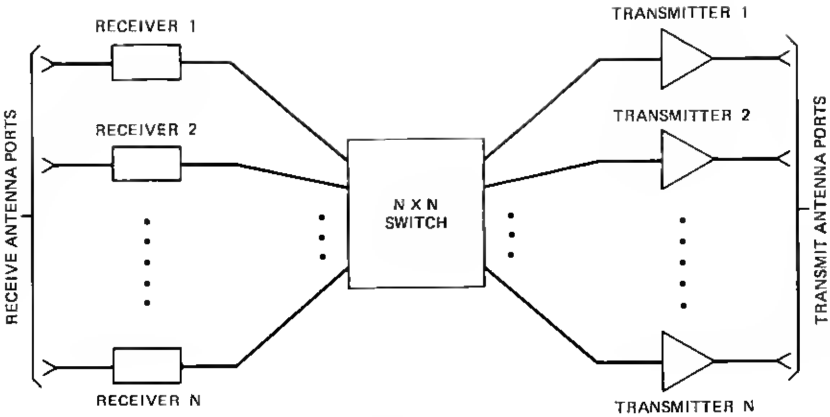


Fig. 1—Satellite transponder.

An on-board switch is provided to route messages originating within the coverage area of one antenna beam to their destinations in the coverage areas of other beams. Access to the satellite is by time division multiple access (TDMA), and the up and down link frequency bands are the same for all beams. This arrangement is often referred to as satellite-switched TDMA.

If each coverage area is serviced by a single transponder, failure of that transponder eliminates all communications to that area. Thus, reliability requirements are generally much higher than in earlier satellite systems employing frequency-division multiple access global beam coverage wherein failure of a single transponder results in partially reduced traffic handling capability to all users.[5,6]

The higher reliability requirement of multiple-spot-beam satellite systems implies that either (*i*) additional redundant hardware must be provided and/or (*ii*) a more efficient strategy be adopted for deploying existing redundant units. Since the total orbital weight of the satellite is fixed by launch vehicle capability, it is desirable to maintain the weight of redundancy-related equipment at a minimum in order that as much weight as possible be available for active communication transponders. In this paper, several configurations employing cold standby redundancy[7–10] are proposed and explored, and additional weight vs reliability trade-offs are established. Consideration is limited to failures of the final high power traveling wave tube amplifiers (TWTAs) since the relatively high failure rate and large weight of these devices would dominate in a more detailed study. Exact results are provided for single satellite systems; from these, reliability bounds for multiple satellite systems, including an in-orbit spare, are derived for a variety of philosophies for spare satellite utilization.

In Section II, the mathematical formulas pertaining to single satellite reliability employing cold standby $M$-on-$N$ redundancy ($M$ standby spare units for $N$ active units with full interconnectivity) are derived assuming that the cold failure rate is zero. From these, we can find, at each point in time, the probability that $j$ transponders have failed, $0 \leqq j \leqq N$.

These formulas are applied in Section III to a specific satellite concept, appropriate for a Thor-Delta launch, to determine transponder reliability vs capacity trade-offs under a weight constraint. Also investigated in Section III is the interconnectivity trade-off; that is, reliability vs total capacity for conditions other than full interconnectivity.

In Section IV, upper and lower reliability bounds are derived for satellite systems containing an arbitrary number of active satellites plus an identical in-orbit spare. Various approaches to spare satellite utilization, differing in degree of complexity, are explored. Results indicate that significant reliability improvements can be achieved through appropriate utilization of the spare satellite. These results, however, are to be interpreted as indicative of a trend rather than as concrete design tools since the probability of catastrophic failure of an active satellite, requiring complete utilization of the spare satellite, was assumed to be zero in deriving the results.

## II. RELIABILITY FORMULAS

In this section, a model for predicting the reliability of a multibeam communication satellite employing $M$-on-$N$ standby redundancy is developed and analyzed. Complete interconnectivity is assumed, that is, any of the $M$ cold standby redundant units may be substituted for any of the $N$ active units upon failure of one of the latter. The cold standby failure rate is assumed to be zero, as is the failure rate of the interconnecting redundancy switches, and the failure rate for each active unit, $\lambda$, is assumed to be constant. Failures are assumed to be characterized by a Poisson point process. An expression for the probability that $N$ units are operational at any point in time is easily established;[11] this result is extended to find the probability of having $j$ active channels as a function of time, $0 \leqq j \leqq N$.

Let the system state $j$ be the number of operational units at time $t$, $0 \leqq j \leqq N + M$, and let $P_j(t)$ be the probability of finding the system in state $j$ at time $t$. Then, since there are $N$ active units for states $j = N, N + 1, \ldots, N + M$, the following relationships hold for those states:

$$P_{N+M}(t + \Delta t) \cong (1 - N\lambda\Delta t)P_{N+M}(t), \tag{1}$$

$$P_j(t + \Delta t) \cong (1 - N\lambda\Delta t)P_j(t) + N\lambda\Delta t P_{j+1}(t),\ N \leqq j < N + M, \tag{2}$$

where the approximations become exact as $\Delta t \to 0$. Similarly, since there are $j$ active units for states $j = 0, 1, \ldots, N$,

$$P_j(t + \Delta t) \cong (1 - j\lambda\Delta t)P_j(t) + (j + 1)\lambda\Delta t P_{j+1}(t),\ 0 \leqq j < N. \quad (3)$$

Letting $\Delta t \to 0$, the following set of differential equations results for the system state probabilities:

$$\frac{dP_{N+M}}{dt} = -N\lambda P_{N+M}. \quad (4)$$

$$\frac{dP_j}{dt} = -N\lambda P_j + N\lambda P_{j+1},\ N \leqq j < N + M. \quad (5)$$

$$\frac{dP_j}{dt} = -j\lambda P_j + (j + 1)\lambda P_{j+1},\ 0 \leqq j < N. \quad (6)$$

Initial conditions are:

$$P_{N+M}(0) = 1. \quad (7)$$

$$P_j(0) = 0,\ 0 \leqq j < N + M. \quad (8)$$

Solutions of these equations are readily obtained for $N \leqq j \leqq N + M$. The solutions are:

$$P_j(t) = \frac{(N\lambda t)^{N+M-j}e^{-N\lambda t}}{(N + M - j)!},\ N \leqq j \leqq N + M. \quad (9)$$

Solutions for $0 \leqq j < N$ are considerably more difficult and are derived in the Appendix.

The exact solutions are rather cumbersome, provide little physical insight into the benefits of standby redundancy, and require high precision arithmetic to obtain numerical results. Approximate formulas are therefore derived in the Appendix, valid for the region of interest $(N - j)\lambda t/(M + 3) \ll 1$. Letting $Q_j$ be the probability of exactly $j$ surviving operational transponders, we obtain:

$$Q_N = \sum_{j=N}^{N+M} P_j \cong 1 - \left[\frac{M + 2}{M + 2 - N\lambda t}\right] \frac{(N\lambda t)^{M+1}}{(M + 1)!} e^{-N\lambda t}. \quad (10)$$

$$Q_{N-1} = P_{N-1}(t) \cong \left[\frac{M^2 + 5M + 6 + \lambda t}{(M + 2)(M + 3 - \lambda t)}\right] \left[\frac{(N\lambda t)^{M+1}e^{-N\lambda t}}{(M + 1)!}\right]. \quad (11)$$

$$Q_j = P_j(t) = \frac{N!N^M(M + 3)^2(\lambda t)^{M+N-j}e^{-N\lambda t}}{j!(N - j - 1)!(M + 2)!(M + 3 - \lambda t)^{N-j}},\ 0 \leqq j \leqq N - 2. \quad (12)$$

## III. SATELLITE TRANSPONDER WEIGHT AND RELIABILITY

We will now study the reliability vs weight trade-off for a particular satellite concept, appropriate for a Thor-Delta launch weight class. The following assumptions are made:

(*i*) The satellite can support the weight of 11 transponders without any traveling wave tube redundancy. (Normalized weight = 11).

(*ii*) The weight of a traveling wave tube amplifier (including power supply) is equal to one-third the weight of a single transponder (including the weight of prime power sources for that transponder).

(*iii*) Each switch point needed for redundancy has a weight equal to 1/15th the weight of a traveling wave tube amplifier.

These assumptions are consistent with weight and power budget requirements of a system described elsewhere.[12]

From these requirements, we see that the following redundancy arrangements are possible:

(A) 10 active transponders
2 standby TWTAs
Complete redundant interconnectivity
(Normalized weight = 10.9)

(B) 9 active transponders
4 standby TWTAs
Complete redundant interconnectivity
(Normalized weight = 10.7)

(C) 8 active transponders
6 standby TWTAs
Complete redundant interconnectivity
(Normalized weight = 10.5)

(D) 9 active transponders
6 standby TWTAs
Redundancy provided in 2 on 3 arrangement
(Normalized weight = 11.2)

Numerical results have been obtained for TWTA failure rates between 1500 and 6000 fits (one fit is a failure rate of 1 per $10^9$ hours). These extremes are based upon (*i*) actual space experience and life test data for space qualified TWTAs, (*ii*) changes in TWT electrical design objectives such as power rating, operating frequency, and multi-mode capability, and (*iii*) projections for future technology improvements.

Shown in Fig. 2 are the reliability predictions vs time for an 11-transponder satellite with no redundancy. Results are plotted for a failure

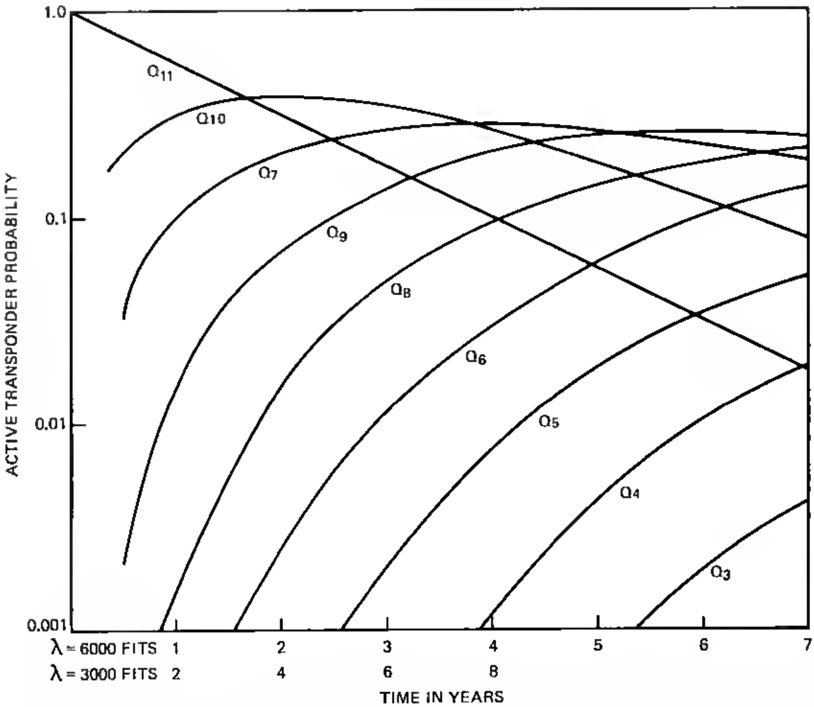


Fig. 2—Active transponder probabilities for a satellite with 11 active transponders and no spare TWTAs. $Q_J$ is the probability that exactly $J$ transponders are operational at time $t, 0 \leqq J \leqq 11$.

rate of 6000 fits. Since the reliability predictions are dependent upon the product, $\lambda t$, predictions can easily be inferred for other failure rates by scaling the time axis by the ratio of $6000/\lambda$, $\lambda$ being the failure rate under consideration. Such a scaling has been performed in Fig. 2 for $\lambda = 3000$ fits.

Results for Cases A–D are shown, respectively, in Figs. 3 through 6. Since Case D employs less than full redundant interconnectivity, we solve equations (4) through (8) for $N = 3$, $M = 2$, and use these to calculate the desired probabilities:

$$\begin{aligned} Q_9 &= P\{3 \text{ active transponders in each of 3 groups}\} \\ &= (Q_3')^3. \end{aligned} \tag{13}$$

$$\begin{aligned} Q_8 = P\{&3 \text{ active transponders in each of 2 groups and} \\ &2 \text{ active transponders in the third group}\} \\ = 3&(Q_3')^2 Q_2'. \end{aligned} \tag{14}$$

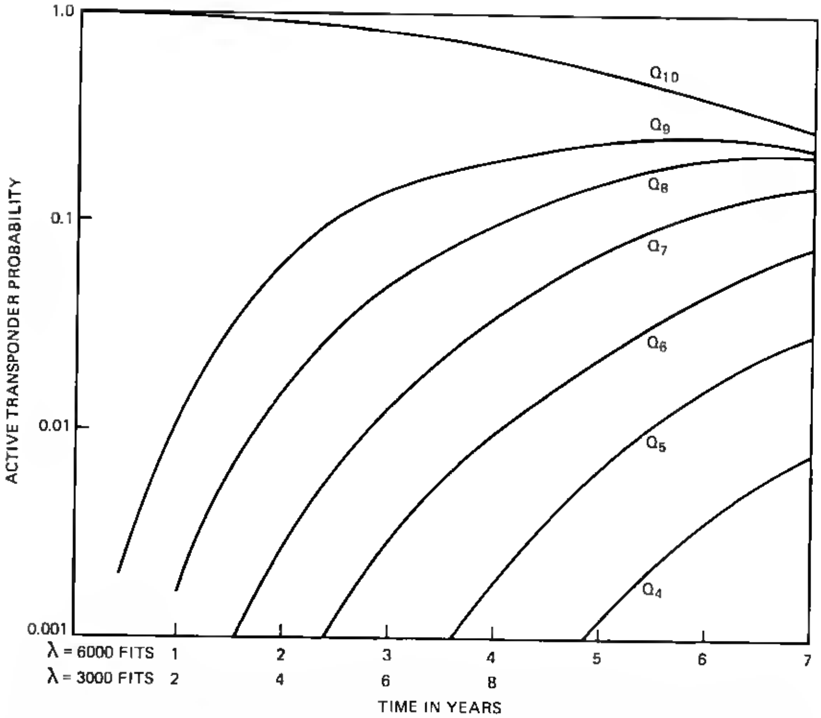


Fig. 3—Active transponder probabilities for a satellite with 10 active transponders and 2 spare TWTAs. Each transponder can access any spare amplifier. $Q_J$ is the probability that exactly $J$ transponders are operational at time $t$, $0 \leq J \leq 10$.

$$
\begin{aligned}
Q_7 = P\{&\text{3 active transponders in each of 2 groups} \\
&\text{and 1 active transponder in the third, or 2} \\
&\text{active transponders in each of 2 groups and} \\
&\text{3 active transponders in the third}\} \\
= {}&3(Q_3')^2Q_1' + 3(Q_2')^2Q_3', \text{ etc.}
\end{aligned}
\tag{15}
$$

Figures (2) through (6) represent a mesh of the exact solutions of the Appendix and the approximate solutions (10) through (12); that is, for large $t$, the exact expressions have been plotted, while, for small $t$, the approximate solutions are used since they provide better accuracy than possible via numerical evaluation of the exact solutions. In all cases, agreement was better than 5 percent at the cross-over point between the exact and approximate solutions.

We now interpret the results appearing in Fig. (2) through (6). A seven-year satellite life is assumed. We see from Fig. 2 that service is quite unreliable in the absence of redundancy. For a TWTA failure rate of 6000 fits, the probability of providing service to all coverage areas is less than 2 percent at the end of satellite life; this figure rises to about

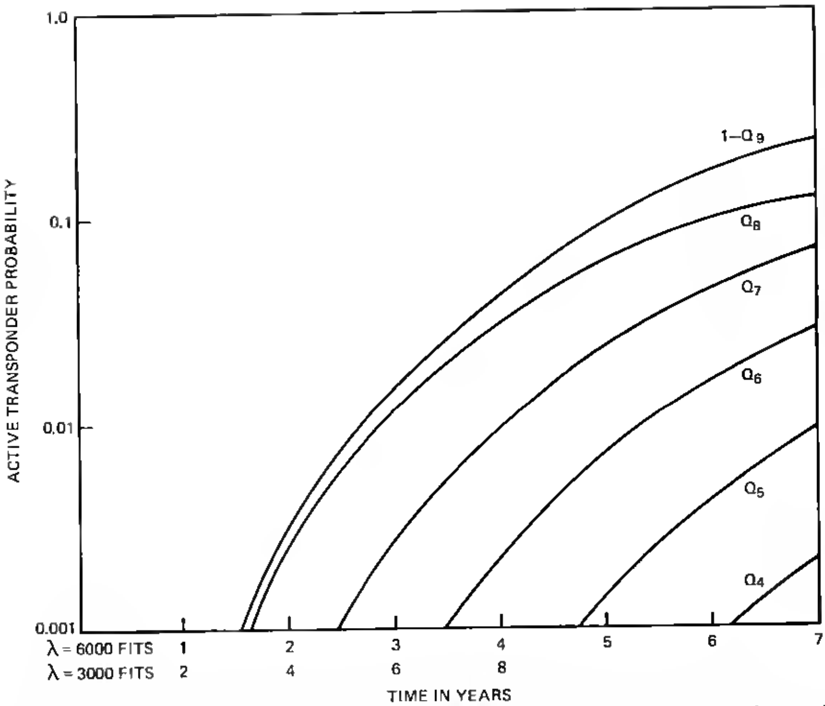


Fig. 4—Active transponder probabilities for a satellite with 9 active transponders and 4 spare TWTAs. Each transponder can access any spare amplifier. $Q_J$ is the probability that exactly $J$ transponders are operational at time $t$, $0 \leqq J \leqq 9$.

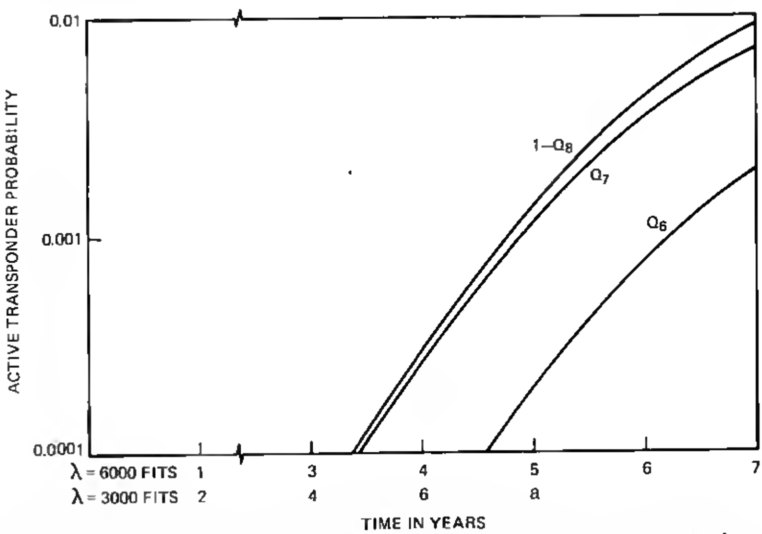


Fig. 5—Active transponder probabilities for a satellite with 8 active transponders and 6 spare TWTAs. Each transponder can access any spare amplifier. $Q_J$ is the probability that exactly $J$ transponders are operational at time $t$, $0 \leqq J \leqq 8$.

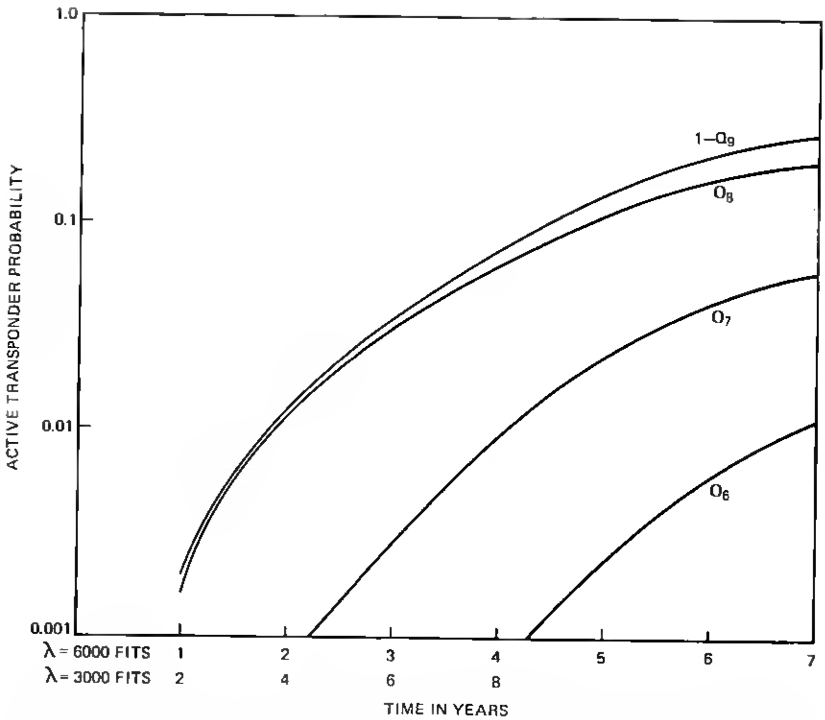


Fig. 6—Active transponder probabilities for a satellite with 9 active transponders and 6 spare TWTAs. Each transponder within a group of 3 can access only the two spare amplifiers assigned to that group. $Q_J$ is the probability that exactly $J$ transponders are operational at time $t$, $0 \leq J \leq 9$.

13 percent for a failure rate of 3000 fits and to 36 percent for $\lambda = 1500$ fits. In fact, the probability of providing service to seven or fewer of the original 11 overage areas is 41 percent for $\lambda = 3000$ fits. Performance is unacceptable for a satellite system employing multiple spot beams, and this case will not receive any further attention.

The expected number of surviving transponders for Cases A through D are plotted in Fig. 7 as functions of time. From these curves we conclude that Case A (10 active transponders, 2 spares, full interconnectivity) provides the maximum number of expected transponder years. From Fig. 3, we see that for this case, the probability of losing at least one transponder by satellite end-of-life is 72 percent for $\lambda = 6000$ fits, 26 percent for $\lambda = 3000$ fits, and 6 percent for $\lambda = 1500$ fits. However, the probability of having seven or more active transponders at satellite end-of-life is 88 percent for $\lambda = 6000$ fits, 99 percent for $\lambda = 3000$ fits, and 99.99 percent for $\lambda = 1500$ fits. Thus, such an arrangement is probably unacceptable for multiple spot-beam satellites, but is a prime candidate for a global coverage satellite since it provides maximum ex-

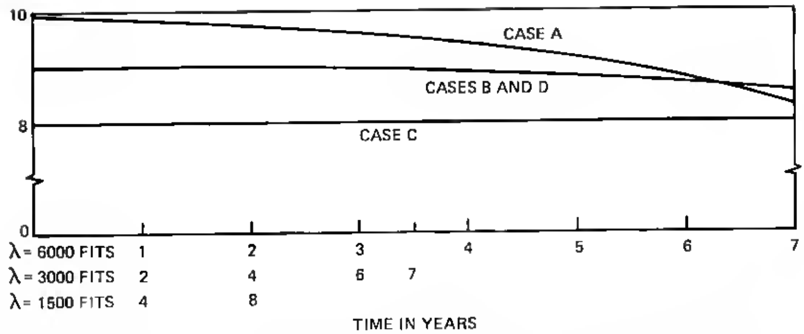


Fig. 7—Expected number of active transponders vs time. Cases A, B, C, and D are as defined in the text.

pected transponder years with high probability of providing at least 70-percent throughput at satellite end-of-life.

For a multiple spot-beam satellite, we see that Case C provides the most dependable service, but that the expected number of transponder years is lower than that for Cases B or D. For Case B, we see that for $\lambda = 6000$ fits, the probability of at least one transponder failing has risen to 24 percent by end of satellite life, and that the probability that no more than one transponder fails is only 88 percent. However, for $\lambda = 3000$ fits, Case B provides for complete service with a 97-percent probability at end of satellite life; the probability that no more than one transponder fails is 99 percent. For $\lambda = 1500$ fits, the corresponding probabilities are 99.9 percent and 99.99 percent. Thus, for a TWTA failure rate of 1500 fits, Case B appears superior to Case C in that an additional transponder is provided with high probability over the life of the satellite. Conversely, for a high TWTA failure rate (6000 fits), Case C is to be preferred since it provides for more dependable service toward the satellite end-of-life.

It is noted further that for a multiple spot-beam satellite, Case B is marginally preferable to Case D if the probability of providing complete service over the life of the satellite is to be maximized (97 percent vs 95 percent for $\lambda = 3000$ fits). Case D, however, provides for a smaller probability that more than one transponder fails (0.5 percent vs 1 percent). Since Case B requires twice the number of switch points to provide for complete interconnectivity, its marginal advantage might disappear if the switch reliability were to be included in the analysis. Conversely, Case D requires slightly more weight than Case B. Thus, for the particular parameters selected for this study, Case B and D are virtually indistinguishable; both provide for highly reliable service over the life of the satellite if the TWTA failure rate is sufficiently small.

We conclude this section with a plot of satellite capacity vs end-of-life reliability for each case discussed (satellite weight constraint). Fig. 8a

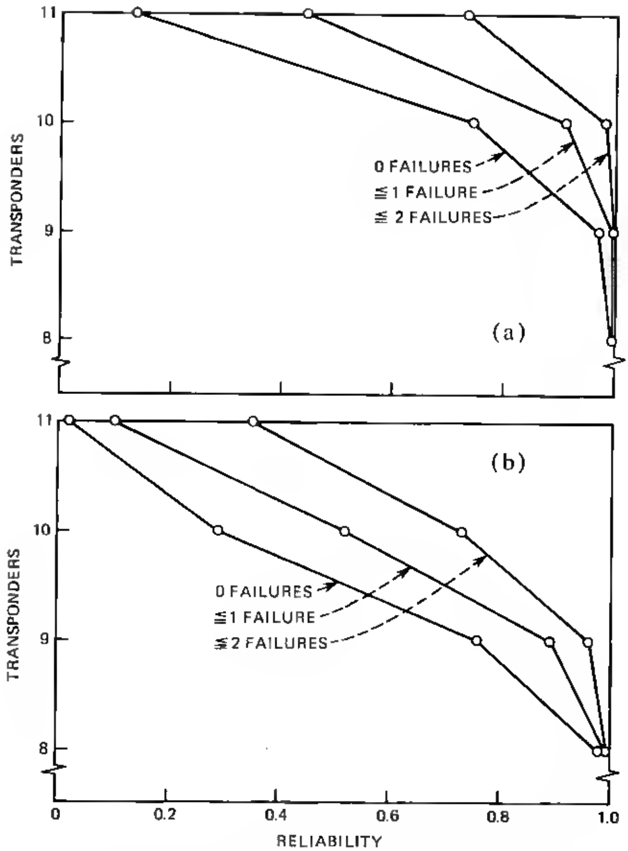


Fig. 8—Probability of success after 7 years vs the original number of active transponders. Success is defined in one of three ways: no failures, no more than one failure, or no more than two failures. (a) $\lambda$ = 3000 fits. (b) $\lambda$ = 6000 fits.

is applicable for $\lambda$ = 3000 fits; Fig. 8b for $\lambda$ = 6000 fits. For these plots, reliability is defined as having no more than 0, 1, or 2 failed transponders.

## IV. APPLICATION TO MULTI-SATELLITE SYSTEMS

We now apply the results of Sections II and III to determine bounds on the success probability, $P_s$, of a system containing $S$ identical active satellites. Success is defined as having all design transponders ($S \times N$ total) available at end-of-life. All satellites are assumed to be launched simultaneously.

Obviously, at any point in time, $P_s(t) = Q_N^S(t)$, where $Q_N(t)$ is the probability of all transponders being active on any given satellite at time $t$. Results for the cases studied in Section III appear in Table I.

Table I — Satellite system success probability; unutilized in-orbit spare

| S | Case A | Case B | Case C | Case D |
|---|---|---|---|---|
| | | (a) $\lambda$ = 6000 fits | | |
| 2 | 0.08 | 0.57 | 0.98 | 0.52 |
| 3 | 0.02 | 0.44 | 0.97 | 0.37 |
| 4 | 0.01 | 0.33 | 0.96 | 0.27 |
| | | (b) $\lambda$ = 3000 fits | | |
| 2 | 0.54 | 0.94 | 0.999 | 0.90 |
| 3 | 0.41 | 0.91 | 0.999 | 0.86 |
| 4 | 0.30 | 0.89 | 0.998 | 0.81 |
| | | (c) $\lambda$ = 1500 fits | | |
| 2 | 0.82 | 0.996 | 0.9999 | 0.984 |
| 3 | 0.74 | 0.994 | 0.9999 | 0.976 |
| 4 | 0.67 | 0.992 | 0.9999 | 0.968 |

From Table I, we conclude that, with the exception of Case C, the success probability is unacceptably low at the higher fit rates. For $\lambda$ = 1500 fits, Case B is relatively attractive.

Since, by any measure, the cost associated with total failure of one satellite of a satellite system is prohibitive, satellite systems generally contain an in-orbit spare. In the following, we explore the possibility of using this spare to improve reliability of systems containing $S$ active satellites plus an identical spare.

Suppose we use the spare satellite by assigning to it all the traffic of the first active satellite to experience a transponder failure. Upon such a failure, each ground station permanently reroutes all communications with the failed satellite to the spare. From eq. (9), the probability $r(t)dt$ that any one of the active satellites loses its first transponder in $[t, t + dt]$, given that the first loss occurs somewhere within $[0, T]$, is:

$$r(t)dt = \frac{N\lambda \dfrac{(N\lambda t)^M e^{-N\lambda t}}{M!} dt}{\displaystyle\int_0^T N\lambda \frac{(N\lambda t)^M e^{-N\lambda t}}{M!} dt} = \frac{N\lambda P_N(t)dt}{N\lambda \displaystyle\int_0^T P_N(t)dt}. \tag{16}$$

The probability $u$ that the spare satellite then successfully completes the mission of the failed active satellite is given by:

$$u = \int_0^T r(t)Q_N(T - t)dt, \tag{17}$$

where $Q_N$ is given by eq. (10). Since, for the mission to be successful, the remaining active satellites must be operational at end-of-life, and since the spare, if needed, can be assigned to any of $S$ satellites, we obtain

Table II — Satellite system success probability. Spare satellite replaces first active to fail

| No. Active Satellites | Case A $\lambda = 1500$ fits | Case A $\lambda = 3000$ fits | Case B $\lambda = 3000$ fits | Case B $\lambda = 6000$ fits | Case C $\lambda = 6000$ fits |
|---|---|---|---|---|---|
| 1 | 0.975 | 0.961 | 0.997 | 0.970 | 0.999 |
| 2 | 0.938 | 0.872 | 0.993 | 0.883 | 0.996 |
| 3 | 0.903 | 0.762 | 0.988 | 0.773 | 0.993 |
| 4 | 0.866 | 0.621 | 0.983 | 0.659 | 0.989 |

$$P_S = Q_N^S + SN\lambda u Q_N^{S-1} \int_0^T P_N(t)dt. \tag{18}$$

The evaluation of eq. (17) is straightforward and is omitted. Evaluation of (18) for Cases A, B, and C of Section II yields the results appearing in Table II.

By comparing these results against those of Table I, we see that the availability of the spare satellite has greatly enhanced the probability of mission success. However, for Case A, we again conclude that the success probabilities are too low for systems employing two or more active satellites. In what follows, we explore the advantages of deploying the spare satellite in a different manner.

Suppose the spare satellite is used by assigning individual transponders on the spare satellite, as needed, to replace failed transponders on the active satellites. Each ground station, then, must be capable of dynamically routing that portion of its traffic destined for the coverage areas of the failed transponders to the spare satellite. Since, for this study, the spare satellite is identical to the active satellites, we conclude that the mission is successful if (*i*) the total number of failed transponders on all active satellites is $\leq N$, (*ii*) no two transponders servicing the same footprint fail on the active satellites, and (*iii*) the assigned transponders on the spare satellite survive after activation for the remainder of the mission. From this, we derive upper and lower bounds on the probability of success as follows.

Let $p_i$ be the probability that $i$ transponders have failed on an active satellite; i.e., $p_i = Q_{N-i}$. Then, the number of ways, $W$, that $i_1$ out of $N$ transponders fail on active satellite 1, $i_2$ out of $N$ transponders (covering footprints different from the $i_1$ failed transponders of satellite 1) fail on satellite 2, . . . , and $i_S$ out of $N$ transponders (covering footprints different from the $i_1$ failed transponders of satellite 1 and the $i_2$ failed transponders of satellite 2, etc.) fail on satellite $S$ is given by:

$$W_{i_1,i_2,\ldots,i_S} = \binom{N}{i_1} \times \binom{N-i_1}{i_2} \times \ldots \times \binom{N-i_1-i_2-\ldots-i_{S-1}}{i_S}$$
$$= \frac{N!}{i_1!i_2!\ldots i_S!(N-i_1-i_2-\ldots-i_S)!}, \tag{19}$$

where $\binom{k}{j}$ represents the binomial coefficient $k!/j!(k-j)!$ The total number of outcomes $T$ in which $i_j$ transponders fail on the $j$th satellite, $1 \leqq j \leqq S$, is given by

$$T_{i_1, i_2, \ldots i_S} = \binom{N}{i_1} \times \binom{N}{i_2} \times \ldots \times \binom{N}{i_S}. \tag{20}$$

The probability of mission success is given by

$$P_s = \sum_{i_1, \ldots, i_S} P(s \mid i_1, \ldots, i_S) p_{i_1} p_{i_2} \ldots p_{i_S} \tag{21}$$

Obviously, $P(s \mid i_1, i_2, \ldots, i_S) = 0, i_1 + i_2 + \ldots + i_S > N$, for then the spare satellite has too few transponders to provide complete coverage.

Let us define $B_k$ as the probability that none of $k$ specific transponders on the spare satellite have failed at mission end-of-life, given that all spare TWTAs on the spare satellite are available to these $k$ transponders, and that the remaining $N-k$ transponders on the spare satellite are unactivated. The $k$ active transponders are activated at the beginning of the mission; the probabilities $B_k$ are obtained by solving eqs. (4) through (8) for $N = k$.

Now, for a given set $\{i_j \mid 1 \leqq j \leqq S\}$, all outcomes of failures are equally likely. Thus, assuming that the transponders on the spare satellite needed to take over for failed transponders are available and do not fail, we see that

$$P(S \mid i_1, \ldots, i_S) \leqq \frac{W_{i_1, \ldots, i_S}}{T_{i_1, \ldots, i_S}}. \tag{22}$$

Similarly, by assuming that $i_1 + i_2 + \ldots i_S$ transponders are initially activated on the spare satellite and that spare TWTAs are available only to these transponders, we see that

$$P(S \mid i_1, \ldots, i_S) \geqq \frac{W_{i_1, \ldots, i_S}}{T_{i_1, \ldots, i_S}} B_{i_1 + i_2 + \ldots + i_S}. \tag{23}$$

Thus,

$$\sum_{\substack{i_1, \ldots, i_S \\ i_1 + i_2 + \ldots + i_S \leqq N}} p_{i_1} p_{i_2} \ldots p_{i_S} \frac{(N-i_1)!(N-i_2)! \ldots (N-i_S)!}{(N!)^{S-1}(N - i_1 - i_2 - \ldots - i_S)!} B_{i_1 + i_2 + \ldots + i_S}$$

$$\leqq P_s \leqq \sum_{\substack{i_1, \ldots, i_S \\ i_1 + i_2 + \ldots + i_S \leqq N}} p_{i_1} p_{i_2} \ldots p_{i_S} \frac{(N-i_1)!(N-i_2)! \ldots (N-i_S)!}{(N!)^{S-1}(N - i_1 - i_2 \ldots - i_S)!}. \tag{24}$$

We apply these bounds to Case A for $\lambda = 3000$ fits and to Case B for $\lambda = 6000$ fits. Results appear in Table III. We note that dynamic tran-

Table III — Satellite system success probability. Spare satellite transponders assigned dynamically without priority

| No. Active Satellites | Case A | Case B |
|---|---|---|
| 1 | $0.998 \leqq P_s \leqq 1.0$ | $0.999 \leqq P_s \leqq 1.0$ |
| 3 | $0.946 \leqq P_s \leqq 0.951$ | $0.951 \leqq P_s \leqq 0.953$ |

sponder assignments on the spare satellite have raised the success probability for Cases A and B appreciably.

Another redundancy strategy, applicable to improving the reliability of systems employing two or more active satellites, also evolves about dynamic assignment of the transponders of the spare satellite as required. For this scheme, however, all available spare TWTAs on the active satellite are allocated on a priority basis such as to maximize the probability of success. For example, if there are two active satellites and one has a single failed transponder whose traffic load has been assumed by the spare satellite, and the transponder covering the same footprint should fail on the second active satellite, then any available spare TWTAs on the second active satellite would be assigned to that failed transponder; if such a spare had previously been brought on line to replace a different failed transponder, the spare satellite would then be employed to pick up the traffic of this second transponder. Again, each ground station must dynamically route that portion of its traffic destined for the coverage areas of the failed transponders to the spare satellite.

The analysis of Section II is not directly applicable to this scheme. This analysis, however, can be applied to a slightly modified scheme whereby not only the spare TWTAs, but also those originally assigned to active transponders, are assigned, as needed, on a priority basis to maximize the probability of mission success. Then, the mission is successful, provided that $(i)$ no more than $N$ transponders have failed on the active satellites, and $(ii)$ no needed transponders on the spare have failed. The probability of success, therefore, is upper-bounded by:

$$P_S \leqq \sum_{\substack{i_1, i_2, \ldots, i_S \\ i_1 + \ldots + i_S \leqq N}} p_{i_1} p_{i_2} \ldots p_{i_S}. \tag{25}$$

Assuming, as before, that $i_1 + i_2 + \ldots + i_S$ transponders needed on the spare satellite begin to fail at time $t = 0$ and that all TWTAs on the spare satellite are available to these transponders, we obtain a lower bound as:

$$P_S \geqq \sum_{\substack{i_1, \ldots, i_S \\ i_1 + i_2 + \ldots + i_S \leqq N}} p_{i_1} p_{i_2} \ldots p_{i_S} B_{i_1 + i_2 + \ldots + i_S}. \tag{26}$$

We apply these results to a system containing three active satellites plus a spare. For Case A, with a failure rate of 3000 fits, the success probability exceeds 0.992, while for Case B, with a failure rate of 6000 fits, the success probability exceeds 0.995. For both these examples, the upper bound given by eq. 25 is within 0.02 percent of unity.

## V. CONCLUSION

The concept of satellite-switched TDMA was briefly reviewed, and the need for highly reliable transponders was discussed. Since, in a satellite-switched TDMA system, each coverage area is serviced by a single transponder, failure of that transponder eliminates service to that coverage area.

Reliability formulas were derived for the probability of having $j$ out of $N$ active transponders operational, $0 \leq j \leq N$, as a function of time for a system employing $M$-on-$N$ cold standby redundancy. From these, approximate formulas, valid for highly reliable systems, were found. These results were then applied to study the reliability vs the number of transponder trade-off under a total transponder weight constraint. A specific concept, allowing 11 transponders with no redundancy, was studied. It was assumed that each transponder was equivalent in weight to two TWTAs plus interconnecting redundancy switches. Failure rate for the TWTA was varied between 1500 and 6000 fits.

Results indicated that for $\lambda = 6000$ fits, 6-on-8 redundancy with complete interconnectivity is required to achieve a success probability greater than 99 percent after seven years. Thus, for this case, reliability requirements result in a capacity reduction of 27 percent. For a lower failure rate of 3000 fits, 4-on-9 redundancy with complete interconnectivity would provide for 97.3-percent reliability; the capacity reduction is then 18 percent. For $\lambda = 1500$ fits, 4-on-9 redundancy with complete interconnectivity would provide 99.8-percent reliability.

Various schemes employing a spare satellite to increase the reliability of a system employing $S$ active satellites were proposed and studied. Such a spare satellite is generally provided to protect against catastrophic failure; the probability of catastrophic failure was assumed to be zero. We saw that for a failure rate of 1500 fits, a reliability of 97.5 percent could be achieved for a single active satellite system employing 2-on-10 complete interconnectivity redundancy; the spare was utilized by assuming the entire burden of the active satellite when the first transponder failure occurred. Similarly, a 99.7-percent reliability could be achieved for a 4-on-9 redundancy and a failure rate of 3000 fits. For the former case, reliability falls to under 95 percent when two or more active satellites are contained in the system. By contrast, a 6-on-8 strategy employing complete interconnectivity provides for reliability exceeding 99 percent for $\lambda = 6000$ fits and 3 active satellites.

To improve reliability of the 2-on-10 and 4-on-9 concepts at higher TWT failure rates, a different utilization of the spare satellite was explored. This utilization consisted of dynamically assigning to the spare satellite only the traffic of failed transponders, rather than the entire traffic load of the first active satellite to lose a transponder. Such a scheme would require that each ground terminal be capable of communicating, simultaneously, with all active satellites plus the spare. Then, for a TWTA failure rate of 3000 fits, a system employing a single active satellite with 2-on-10 redundancy plus an identical spare satellite would have a success probability exceeding 99.8 percent. Thus, high reliability is achieved at the expense of 9-percent traffic-handling reduction of the active satellite potential. Under the same conditions, a system containing three active satellites plus a spare would have a success probability between 94.6 percent and 95.1 percent.

To increase the reliability of multiple satellite systems still further, a concept again utilizing dynamic transponder allocation to the spare satellite was proposed. For this concept, however, the TWTAs of the active satellites were assigned among the active transponders on a priority basis to prevent failure when possible. It was then found that a system employing three active satellites plus a spare, each of 2-on-10 redundancy, would have a success probability exceeding 99.2 percent for a TWTA failure rate of 3000 fits; a 3-active plus spare satellite system, each of 4-on-9 redundancy, would have a success probability exceeding 99.5 percent for a TWTA failure rate of 6000 fits.

Thus, we conclude that for the type of satellite-switched TDMA system studied, we must accept a 27-percent reduction of capacity to achieve a 99-percent probability of losing no transponder if we do not utilize the presence of a spare satellite. If the spare satellite is properly employed, and if the probability of catastrophic failure is vanishingly small, we achieve even higher success probabilities at the expense of 9 percent in traffic-handling capability. Since a greater volume of traffic can then be handled, and since the total traffic demand upon the satellite system grows with the number of service areas which can be interconnected at a rate greater than the traffic demand of individual additional areas, we conclude that utilization of the spare satellite, as proposed above, is highly desirable.

## APPENDIX

In this appendix, we obtain exact solutions to eqs. (4) through (6), subject to initial conditions (7) and (8). Approximate solutions are also derived. For $N \leqq j \leqq N + M$, the exact solutions are:

$$P_j(t) = \frac{(N\lambda t)^{N+M-j} e^{-N\lambda t}}{(N + M - j)!}, N \leqq j \leqq N + M. \tag{9}$$

For $0 \leqq j < N$, we solve via Laplace transform techniques. From eq. (6),

$$P_j(s) = \frac{(j+1)\lambda P_{j+1}(s)}{s + j\lambda}. \tag{27}$$

Thus,

$$P_{N-1}(s) = \frac{N\lambda P_N(s)}{s + (N-1)\lambda} \tag{28}$$

$$P_{N-2}(s) = \frac{N(N-1)\lambda^2 P_N(s)}{[s + (N-1)\lambda][s + (N-2)\lambda]} \tag{29}$$

$$\vdots$$

$$P_j(s) = \frac{N!\lambda^{N-j}}{j!} \frac{P_N(s)}{\prod_{k=1}^{N-j} [s + (N-k)\lambda]}. \tag{30}$$

From eq. (9),

$$P_N(s) = \frac{(N\lambda)^M}{(S + N\lambda)^{M+1}}. \tag{31}$$

Thus,

$$P_j(s) = \frac{N!(N\lambda)^M \lambda^{N-j}}{j!} \left[ \frac{1}{(s + N\lambda)^{M+1}} \right] \times \left[ \frac{1}{\prod_{k=1}^{N-j} \{s + (N-k)\lambda\}} \right] \qquad 0 \leqq j < N. \tag{32}$$

The various $P_j$'s are seen to possess a multiple pole of order $M + 1$ at $s = -N\lambda$ and simple poles at $s = -(N-k)\lambda$, $k = 1, \ldots, N - j$. All poles are in the left-half complex $s$-plane. We find $P_j(t)$ by inverting a partial fraction expansion. The result is:

$$P_j(t) = \frac{N!N^M}{j!} e^{-N\lambda t} \sum_{i=1}^{N-j} \frac{1}{i^{M+1} \prod_{\substack{k=1 \\ k \neq i}}^{N-j} (i-k)} \times \left[ e^{i\lambda t} - \sum_{l=0}^{M} \frac{(i\lambda t)^{M-l}}{(M-l)!} \right]. \tag{33}$$

We now obtain approximate formulas for $P_j(t)$, $0 \leqq j \leqq N$. Let

$$I = e^{i\lambda t} - \sum_{l=0}^{M} \frac{(i\lambda t)^{M-l}}{(M-l)!} \tag{34}$$

$$= \sum_{l=M+1}^{\infty} \frac{(i\lambda t)^l}{l!} \tag{35}$$

$$= \frac{(i\lambda t)^{M+1}}{(M+1)!}\left[1 + \frac{i\lambda t}{M+2} + \frac{(i\lambda t)^2}{(M+3)(M+2)} + \ldots\right]. \tag{36}$$

For $i\lambda t/(M+3) \ll 1$,

$$I \cong \frac{(i\lambda t)^{M+1}}{(M+1)!}\left[1 + \frac{i\lambda t}{M+2} + \frac{(i\lambda t)^2}{(M+2)(M+3)} + \frac{(i\lambda t)^3}{(M+2)(M+3)^2} + \ldots\right] \tag{37}$$

$$\cong \frac{(i\lambda t)^{M+1}}{(M+1)!}\left[\frac{M+3}{M+2}\right]\left[\frac{M+2}{M+3} + \frac{i\lambda t}{M+3} + \frac{(i\lambda t)^2}{(M+3)^2} + \frac{(i\lambda t)^3}{(M+3)^3} + \ldots\right] \tag{38}$$

$$\cong \frac{(i\lambda t)^{M+1}}{(M+1)!}\left[\frac{M+3}{M+2}\right]\left[-\frac{1}{M+3} + 1 + \frac{i\lambda t}{M+3} + \frac{(i\lambda t)^2}{(M+3)^2} + \ldots\right] \tag{39}$$

$$\cong \frac{(i\lambda t)^{M+1}}{(M+1)!}\left[\frac{M+3}{M+2}\right]\left[\frac{1}{1 - \dfrac{i\lambda t}{M+3}} - \frac{1}{M+3}\right]. \tag{40}$$

Now, it is readily shown that

$$\frac{1}{\prod\limits_{\substack{k=1\\k\neq i}}^{N-j} (i-k)} = \frac{(-1)^{N-j-i}}{(N-j-i)!(i-1)!}. \tag{41}$$

Thus,

$$P_j(t) \cong \frac{N!N^M e^{-N\lambda t}}{j!} \sum_{i=1}^{N-j} \frac{(-1)^{N-j-i}}{(N-j-i)!(i-1)!i^{M+1}} \times \left[\frac{(i\lambda t)^{M+1}}{(M+1)!}\right]\left[\frac{M+3}{M+2}\right]\left[\frac{1}{1 - \dfrac{i\lambda t}{M+3}} - \frac{1}{M+3}\right]. \tag{42}$$

For $\lambda t/(M+3) \ll 1$,

$$\frac{1}{1-\dfrac{i\lambda t}{M+3}} \cong \left[\frac{1}{1-\dfrac{\lambda t}{M+3}}\right]^i. \tag{43}$$

Thus,

$$P_j(t) \cong \frac{N!N^M e^{-N\lambda t}(-1)^{N-j}}{j!} \sum_{i=1}^{N-j} (-1)^i i \binom{N-j}{i} \times \left[\left(\frac{1}{1-\dfrac{\lambda t}{M+3}}\right)^i - \frac{1}{M+3}\right]. \tag{44}$$

Let

$$S = \sum_{i=1}^{N-j} (-1)^i i \binom{N-j}{i} \left(\frac{1}{1-\dfrac{\lambda t}{M+3}}\right)^i. \tag{45}$$

Now, for any $\beta$,

$$(1-\beta)^{N-j} = \sum_{i=0}^{N-j} \binom{N-j}{i} (-\beta)^i \tag{46}$$

$$\frac{d}{d\beta}\left[(1-\beta)^{N-j}\right] = -(N-j)(1-\beta)^{N-j-1} \tag{47}$$

$$= \sum_{i=0}^{N-j} -i \binom{N-j}{i} (-\beta)^{i-1} \tag{48}$$

$$= \frac{1}{\beta} \sum_{i=0}^{N-j} \binom{N-j}{i} i(-\beta)^i \tag{49}$$

$$= \frac{1}{\beta}\left[S \,\middle|\, \beta = \frac{1}{1-\dfrac{\lambda t}{M+3}}\right]. \tag{50}$$

Thus,

$$S = -\left[\frac{1}{1-\dfrac{\lambda t}{M+3}}\right](N-j)\left[1-\frac{1}{1-\dfrac{\lambda t}{M+3}}\right]^{N-j-1}. \tag{51}$$

Now,

$$\sum_{i=1}^{N-j} (-1)^i i \binom{N-j}{i} = S\Big|_{\beta=1} \tag{52}$$

$$= \begin{cases} 0, j \leqq N-2 \\ -1, j = N-1 \end{cases}. \tag{53}$$

Thus,

$$P_{N-1}(t) \cong \left[\frac{M^2 + 5M + 6 + \lambda t}{(M+2)(M+3-\lambda t)}\right]\left[\frac{(N\lambda t)^{M+1} e^{-N\lambda t}}{(M+1)!}\right] \tag{54}$$

$$P_j(t) \cong \frac{N! N^M (M+3)^2 (\lambda t)^{M+N-j} e^{-N\lambda t}}{j!(N-j-1)!(M+2)!(M+3-\lambda t)^{N-j}}\ 0 \leqq j \leqq N-2. \tag{55}$$

Finally, we obtain an approximate formula for the probability $Q_N$ that all $N$ transponders are active at time $t$. We note that

$$Q_N(t) = \sum_{j=N}^{N+M} P_j(t) \tag{56}$$

$$= e^{-N\lambda t} \sum_{j=N}^{N+M} \frac{(N\lambda t)^{N+M-j}}{(N+M-j)!} \tag{57}$$

$$= e^{-N\lambda t}\left[e^{N\lambda t} - \sum_{k=M+1}^{\infty} \frac{(N\lambda t)^k}{k!}\right]. \tag{58}$$

For $N\lambda t/(M+2) \ll 1$,

$$Q_N(t) \cong e^{-N\lambda t}\left\{e^{N\lambda t} - \frac{(N\lambda t)^{M+1}}{(M+1)!} \times \left[1 + \frac{N\lambda t}{M+2} + \frac{(N\lambda t)^2}{(M+2)^2} + \dots\right]\right\} \tag{59}$$

$$\cong 1 - \frac{(N\lambda t)^{M+1} e^{-N\lambda t}}{(M+1)!}\left[\frac{1}{1 - \dfrac{N\lambda t}{M+2}}\right]. \tag{60}$$

Thus,

$$Q_N(t) \cong 1 - \left[\frac{M+2}{M+2-N\lambda t}\right]\frac{(N\lambda t)^{M+1}}{(M+1)!} e^{-N\lambda t}. \tag{61}$$

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